Midterm 2 — March 15, 2017   Duration: 50 minutes
*This test has **5 questions** on **6 pages**, for a total of 25 points.*

- Read all the questions carefully before starting to work.

- All questions are long-answer; you should give complete arguments and explanations for all your calculations; answers without justifications will not be marked.

- Continue on the back of the previous page if you run out of space. If you continue a problem on a different page, indicate this clearly at the bottom of the problem's original page.

- This is a closed-book examination. **None of the following are allowed**: documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, smart watches etc.)

First Name: _____   Last Name: _____

Student-No: _____   Section: **201** _____

Signature: _____

Question:	1	2	3	4	5	Total
Points:	5	7	5	5	3	25
Score:						

2 marks   1. (a) For what values of the constant a is the vector field

$$\vec{F} = \underbrace{(3x^2 + 3y^2 \cos(x))}_{P}\vec{i} + \underbrace{(ay \sin(x) + 5y^3)}_{Q}\vec{j}$$

conservative?

The domain of $\vec{F}$ is simply connected, so we have to check $P_y = Q_x$:

$$\left. \begin{array}{l} P_y = 6y \cos(x) \\ Q_x = ay \cos(x) \end{array} \right\} \Rightarrow \underline{a = 6} \text{ is the only value that}$$

makes $\vec{F}$ conservative.

3 marks   (b) For the values of a determined in the first part of the question compute the integral

$$\int_C \vec{F} \cdot d\vec{r},$$

where C is the semicircle with center $(0, \frac{5}{2})$, radius $\frac{3}{2}$ and $x \leq 0$ oriented counter-clockwise.

$\vec{F}$ is conservative. Need to find a potential function f.

$$f_x \overset{!}{=} P = 3x^2 + 3y^2 \cos(x) \Rightarrow f = x^3 + 3y^2 \sin(x) + g(y)$$

$$f_y = 6y \sin(x) + g'(y) \; -$$

7 marks   2. Compute $\int_C \vec{F}\,d\vec{r}$, where

$$\vec{F} = \langle 3x^2 y + y^3 + 3e^x, 3e^{y^2} + 9 \rangle$$

and C is the semicircle in the upper half-plane from $(-2,0)$ to $(2,0)$. [Hint: Do not attempt to calculate the integral directly.]

4 marks   3. (a) Let $\vec{\mathbf{F}} = P\vec{\imath} + Q\vec{\jmath} + R\vec{\mathbf{k}}$ be a vector field and f a function. Fill in the blanks to make the following formula correct

$$\text{curl}(f\vec{\mathbf{F}}) = \underline{\quad} \times \vec{\mathbf{F}} + f\underline{\quad}(\vec{\mathbf{F}}).$$

Show your work to justify your answer.

1 mark   (b) Let $\vec{\mathbf{F}}_1 = P_1\vec{\imath} + Q_1\vec{\jmath} + R_1\vec{\mathbf{k}}$ and $\vec{\mathbf{F}}_2 = P_2\vec{\imath} + Q_2\vec{\jmath} + R_2\vec{\mathbf{k}}$ be two vector fields. What is

$$\text{div}(\text{curl}(\vec{\mathbf{F}}_1 \times \vec{\mathbf{F}}_2))?$$

5 marks   4. Parametrize the part of the ellipsoid $x^2 + 2y^2 + 2z^2 = 1$ that lies inside the paraboloid $x = \sqrt{3}y^2 + \sqrt{3}z^2$.

3 marks   5. Find an equation for the tangent plane to the surface given by

$$\vec{r}(u, v) = \langle u^2, \, 2u \sin v, \, u \cos v \rangle$$

at the point $\vec{r}(1, 0)$.