

Midterm 2 — March 15, 2017 Duration: 50 minutes*This test has 5 questions on 6 pages, for a total of 25 points.*

- Read all the questions carefully before starting to work.
- All questions are long-answer; you should give complete arguments and explanations for all your calculations; answers without justifications will not be marked.
- Continue on the back of the previous page if you run out of space. If you continue a problem on a different page, indicate this clearly at the bottom of the problem's original page.
- This is a closed-book examination. **None of the following are allowed:** documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, smart watches etc.)

First Name: _____ Last Name: _____

Student-No: _____ Section: **201** _____

Signature: _____

Question:	1	2	3	4	5	Total
Points:	5	7	5	5	3	25
Score:						

Student Conduct during Examinations

- Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
- Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
- No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
- Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
- Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
 - speaking or communicating with other examination candidates, unless otherwise authorized;
 - purposely exposing written papers to the view of other examination candidates or imaging devices;
 - purposely viewing the written papers of other examination candidates;
 - using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
 - using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
- Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
- Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
- Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

2 marks

1. (a) For what values of the constant
- a
- is the vector field

$$\vec{F} = \underbrace{(3x^2 + 3y^2 \cos(x))}_{P} \vec{i} + \underbrace{(ay \sin(x) + 5y^3)}_{Q} \vec{j}$$

conservative?

The domain of \vec{F} is simply connected, so we have to check $P_y = Q_x$:

$$\left. \begin{array}{l} P_y = 6y \cos(x) \\ Q_x = ay \cos(x) \end{array} \right\} \Rightarrow \underline{a=6} \text{ is the only value that makes } \vec{F} \text{ conservative.}$$

3 marks

- (b) For the values of
- a
- determined in the first part of the question compute the integral

$$\int_C \vec{F} \cdot d\vec{r},$$

where C is the semicircle with center $(0, \frac{5}{2})$, radius $\frac{3}{2}$ and $x \leq 0$ oriented counter-clockwise.

\vec{F} is conservative. Need to find a potential function f .

$$f_x \stackrel{!}{=} P = 3x^2 + 3y^2 \cos(x) \Rightarrow f = x^3 + 3y^2 \sin(x) + g(y)$$

$$f_y = 6y \sin(x) + g'(y) -$$

7 marks

2. Compute $\int_C \vec{\mathbf{F}} d\vec{\mathbf{r}}$, where

$$\vec{\mathbf{F}} = \langle 3x^2y + y^3 + 3e^x, 3e^{y^2} + 9 \rangle$$

and C is the semicircle in the upper half-plane from $(-2, 0)$ to $(2, 0)$. [Hint: Do not attempt to calculate the integral directly.]

4 marks

3. (a) Let $\vec{\mathbf{F}} = P\vec{\mathbf{i}} + Q\vec{\mathbf{j}} + R\vec{\mathbf{k}}$ be a vector field and f a function. Fill in the blanks to make the following formula correct

$$\text{curl}(f\vec{\mathbf{F}}) = \underline{\hspace{1cm}} \times \vec{\mathbf{F}} + f \underline{\hspace{1cm}}(\vec{\mathbf{F}}).$$

Show your work to justify your answer.

1 mark

- (b) Let $\vec{\mathbf{F}}_1 = P_1\vec{\mathbf{i}} + Q_1\vec{\mathbf{j}} + R_1\vec{\mathbf{k}}$ and $\vec{\mathbf{F}}_2 = P_2\vec{\mathbf{i}} + Q_2\vec{\mathbf{j}} + R_2\vec{\mathbf{k}}$ be two vector fields. What is

$$\text{div}(\text{curl}(\vec{\mathbf{F}}_1 \times \vec{\mathbf{F}}_2))?$$

5 marks

4. Parametrize the part of the ellipsoid $x^2 + 2y^2 + 2z^2 = 1$ that lies inside the paraboloid $x = \sqrt{3}y^2 + \sqrt{3}z^2$.

3 marks

5. Find an equation for the tangent plane to the surface given by

$$\vec{\mathbf{r}}(u, v) = \langle u^2, 2u \sin v, u \cos v \rangle$$

at the point $\vec{\mathbf{r}}(1, 0)$.