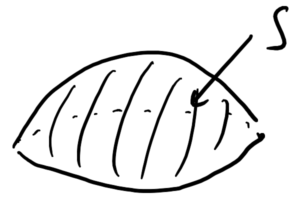
Problem 9. (**10 points.**) Let S be the hemisphere $\{x^2+y^2+z^2=1, z \geq 0\}$ oriented with $\mathbf{N}$ pointing away from the origin. Evaluate the flux integral

$$\iint_S \mathbf{F}\cdot d\mathbf{S}$$

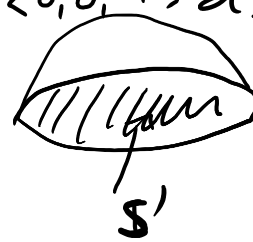
where

$$\mathbf{F} = (x+\cos(z^2))\mathbf{i} + (y+\overset{\sin}{\cancel{\ln}}(x^2+z^5))\mathbf{j} + \sqrt{x^2+y^2}\,\mathbf{k}.$$

$$\operatorname{div} F = 1+1+0 = 2$$



$$\underbrace{\iiint_{\substack{\text{upper half}\\ \text{ball}}} 2\,dx\,dy\,dz}_{2\cdot\frac{1}{2}\mathrm{Vol}(\text{ball of } r=1)} = \underbrace{\iint_S \vec{F}\cdot\vec{N}\,dS}_{?} + \underbrace{\iint_{S'} \vec{F}\cdot\langle 0,0,-1\rangle\,dS}_{\text{easy to compute}}$$



$$= \frac{4\pi}{3}$$

$$-\iint_{S'} \sqrt{x^2+y^2}\,dS = -\int_{\vartheta=0}^{2\pi}\int_{r=0}^{1} r\cdot r\,dr\,d\vartheta = -2\pi\cdot\frac{1}{3}r^3\Big|_0^1 = -\frac{2\pi}{3}$$

$$? = \frac{4\pi}{3} - \left(-\frac{2\pi}{3}\right) = \frac{6\pi}{3} = \underline{\underline{2\pi}}$$