First Name: ______________________ Last Name: ______________________

Student-No: ______________________ Section: ______________________

Very short answer questions

1. [2 marks] Each part is worth 1 mark. Please write your answers in the boxes. **Marking scheme:** 1 for each correct, 0 otherwise

   (a) Evaluate $\tan\left(\frac{\pi}{3}\right)$.

   Answer: $\sqrt{3}$

   Solution:

   $$\sin \pi/3 = \frac{\sqrt{3}}{2} \qquad \cos \pi/3 = \frac{1}{2} \qquad \text{so } \tan \pi/3 = \sqrt{3}$$

   Else draw the appropriate $2 : 1 : \sqrt{3}$ triangle.

   (b) Compute $\lim_{t \to -1} \left(\frac{t-2}{t+3}\right)$.

   Answer: $-3/2$

   Solution:

   $$\lim_{t \to -1} \left(\frac{t-2}{t+3}\right) = \frac{\lim_{t \to -1}(t-2)}{\lim_{t \to -1}(t+3)} = -3/2.$$

Short answer questions — you must show your work

2. [4 marks] Each part is worth 2 marks.

   (a) Find all solutions to $x^3 - 3x^2 - x + 3 = 0$

   Solution:

   $$x^3 - 3x^2 - x + 3 = x^2(x-3) - (x-3) = (x^2-1)(x-3) = (x-1)(x+1)(x-3)$$

   So solutions are $x = -1, 1, 3$. **Marking scheme:** If all 3 soln then 2 marks. If some factoring and 1 or 2 solutions, then 1 mark. If 1 or 2 solution and no working, then 0. If no solutions then 0. ALSO — if a student factors correctly but then gets signs wrong on roots or does not give roots, then give 1 mark.

   (b) Compute the limit $\lim_{x \to 2} \frac{x-2}{x^2-4}$

Solution: If try naively then we get 0/0, so we simplify first:

$$\frac{x-2}{x^2-4} = \frac{x-2}{(x-2)(x+2)} = \frac{1}{x+2}$$

Hence the limit is $\lim_{x\to 2} \frac{1}{x+2} = 1/4$. **Marking scheme:** 1 for factoring+cancelling, 1 for answer. If answer with no working then 0.

Long answer question — you must show your work

3. [4 marks] Compute the limit $\lim_{x\to 1} \frac{\sqrt{x+2}-\sqrt{4-x}}{x-1}$.

Solution: If we try to do the limit naively we get 0/0. Hence we must simplify. **Marking scheme:** If multiply by correct conjugate then 2 marks. If multiply by something close to correct then 1. Else 0.

$$\begin{aligned}
\frac{\sqrt{x+2}-\sqrt{4-x}}{x-1} &= \frac{\sqrt{x+2}-\sqrt{4-x}}{x-1} \cdot \frac{\sqrt{x+2}+\sqrt{4-x}}{\sqrt{x+2}+\sqrt{4-x}} \\
&= \frac{(x+2)-(4-x)}{(x-1)(\sqrt{x+2}+\sqrt{4-x})} \\
&= \frac{2x-2}{(x-1)(\sqrt{x+2}+\sqrt{4-x})} \\
&= \frac{2}{\sqrt{x+2}+\sqrt{4-x}}
\end{aligned}$$

Marking scheme: If correct simplification then 1 mark, else 0. So the limit is

$$\begin{aligned}
\lim_{x\to 1} \frac{\sqrt{x+2}-\sqrt{4-x}}{x-1} &= \lim_{x\to 1} \frac{2}{\sqrt{x+2}+\sqrt{4-x}} \\
&= \frac{2}{\sqrt{3}+\sqrt{3}} \\
&= \frac{1}{\sqrt{3}}
\end{aligned}$$

Marking scheme: 1 for answer.

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Very short answer questions

1. 2 marks Each part is worth 1 marks. Please write your answers in the boxes. **Marking scheme:** 1 for each correct, 0 otherwise

(a) Compute $\tan\left(\frac{\pi}{6}\right)$.

Answer: $\frac{1}{\sqrt{3}}$

Solution:

$$\cos\pi/6 = \frac{\sqrt{3}}{2} \qquad \sin\pi/6 = \frac{1}{2} \qquad \text{so } \tan\pi/3 = \frac{1}{\sqrt{3}}$$

Else draw the appropriate $2:1:\sqrt{3}$ triangle.

(b) Compute $\lim_{t\to-2}\left(\frac{t-5}{t+4}\right)$.

Answer: $-7/2$

Solution:

$$\lim_{t\to-2}\left(\frac{t-5}{t+4}\right) = \frac{\lim_{t\to-2}(t-5)}{\lim_{t\to-2}(t+4)} \qquad = -7/2.$$

Short answer questions — you must show your work

2. 4 marks Each part is worth 2 marks.

(a) Find all solutions to $x^3 - x^2 - 4x + 4 = 0$

Solution:

$$x^3 - x^2 - 4x + 4 = x^2(x-1) - 4(x-1) = (x^2-4)(x-1) = (x-2)(x+2)(x-1)$$

So solutions are $x = 2, -2, 1$. **Marking scheme:** If all 3 soln then 2 marks. If some factoring and 1 or 2 solutions, then 1 mark. If 1 or 2 solution and no working, then 0. If no solutions then 0.

(b) Compute the limit $\lim_{x\to3}\frac{x-3}{x^2-9}$

Solution: If try naively then we get 0/0, so we simplify first:

$$\frac{x-3}{x^2-9} = \frac{x-3}{(x-3)(x+3)} = \frac{1}{x+3}$$

Hence the limit is $\lim_{x\to 3} \frac{1}{x+3} = 1/6$. **Marking scheme:** 1 for factoring+cancelling, 1 for answer. If answer with no working then 0.

Long answer question — you must show your work

3. 4 marks Compute the limit $\lim_{x\to 3} \frac{\sqrt{x-2}-\sqrt{4-x}}{x-3}$.

Solution: If we try to do the limit naively we get 0/0. Hence we must simplify. **Marking scheme:** If multiply by correct conjugate then 2 marks. If multiply by something close to correct then 1. Else 0.

$$\begin{aligned}\frac{\sqrt{x-2}-\sqrt{4-x}}{x-3} &= \frac{\sqrt{x-2}-\sqrt{4-x}}{x-3} \cdot \frac{\sqrt{x-2}+\sqrt{4-x}}{\sqrt{x-2}+\sqrt{4-x}} \\ &= \frac{(x-2)-(4-x)}{(x-3)(\sqrt{x-2}+\sqrt{4-x})} \\ &= \frac{2x-6}{(x-3)(\sqrt{x-2}+\sqrt{4-x})} \\ &= \frac{2}{\sqrt{x-2}+\sqrt{4-x}}\end{aligned}$$

Marking scheme: If correct simplification then 1 mark, else 0. So the limit is

$$\begin{aligned}\lim_{x\to 3}\frac{\sqrt{x-2}-\sqrt{4-x}}{x-3} &= \lim_{x\to 3}\frac{2}{\sqrt{x-2}+\sqrt{4-x}} \\ &= \frac{2}{1+1} \\ &= 1.\end{aligned}$$

Marking scheme: 1 for answer.

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Very short answer questions

1. 2 marks Each part is worth 1 mark. Please write your answers in the boxes. **Marking scheme:** 1 for each correct, 0 otherwise

   (a) Evaluate $\csc\left(\frac{\pi}{3}\right)$.

   Answer: $\frac{2}{\sqrt{3}}$

   Solution:

   $$\sin \pi/3 = \frac{\sqrt{3}}{2} \qquad \csc\theta = \frac{1}{\sin\theta} \qquad \text{so } \csc\pi/3 = \frac{2}{\sqrt{3}}$$

   Else draw the appropriate $2 : 1 : \sqrt{3}$ triangle.

   (b) Compute $\lim_{t\to-1}\left(\frac{t^2}{t-1}\right)$.

   Answer: $-1/2$

   Solution:

   $$\lim_{t\to-1}\left(\frac{t^2}{t-1}\right) = \frac{\lim_{t\to-1}(t^2)}{\lim_{t\to-1}(t-1)} \qquad = -1/2.$$

Short answer questions — you must show your work

2. 4 marks Each part is worth 2 marks.

   (a) Let $f(x) = 3x^2-7x-3$ and $g(x) = 2x^2-6x+3$. Find all values of x for which $f(x) = g(x)$.

   Solution:

   $$3x^2 - 7x - 3 = 2x^2 - 6x + 3 \qquad \Leftrightarrow x^2 - x - 6 = 0 \qquad \Leftrightarrow (x-3)(x+2) = 0$$

   So solutions are $x = 3, -2$. **Marking scheme:** If both soln then 2 marks. If only one soln: 1 mark with work, 0 marks with no work. If correct work but wrong signs on solutions, 1 mark. If set equal to each other and tried to solve but couldn't, 1 mark. A carefully drawn graph counts as work, but a sloppy or careless graph does not.

   (b) Compute the limit $\lim_{x\to-2}\frac{x+2}{x^2-4}$

Solution: If try naively then we get $0/0$, so we simplify first:

$$\frac{x+2}{x^2-4} = \frac{x+2}{(x-2)(x+2)} = \frac{1}{x-2}$$

Hence the limit is $\lim_{x\to-2} \frac{1}{x-2} = -1/4$. **Marking scheme:** 1 for factoring+cancelling, 1 for answer. If answer with no working then 0. Correct except for sign, 1 mark.

Long answer question — you must show your work

3. [4 marks] Compute the limit $\lim_{x\to 1} \frac{\sqrt{3x+5}-\sqrt{2x+6}}{x-1}$.

Solution: If we try to do the limit naively we get $0/0$. Hence we must simplify. **Marking scheme:** If multiply by correct conjugate then 2 marks. If multiply by something close to correct then 1. Else 0.

$$\begin{aligned}\frac{\sqrt{3x+5}-\sqrt{2x+6}}{x-1} &= \frac{\sqrt{3x+5}-\sqrt{2x+6}}{x-1} \cdot \frac{\sqrt{3x+5}+\sqrt{2x+6}}{\sqrt{3x+5}+\sqrt{2x+6}} \\ &= \frac{(3x+5)-(2x+6)}{(x-1)(\sqrt{3x+5}+\sqrt{2x+6})} \\ &= \frac{x-1}{(x-1)(\sqrt{3x+5}+\sqrt{2x+6})} \\ &= \frac{1}{\sqrt{3x+5}+\sqrt{2x+6}}\end{aligned}$$

Marking scheme: If correct simplification then 1 mark, else 0. So the limit is

$$\begin{aligned}\lim_{x\to 1} \frac{\sqrt{3x+5}-\sqrt{2x+6}}{x-1} &= \lim_{x\to 1} \frac{1}{\sqrt{3x+5}+\sqrt{2x+6}} \\ &= \frac{1}{\sqrt{8}+\sqrt{8}} \\ &= \frac{1}{2\sqrt{8}} = \frac{1}{4\sqrt{2}}\end{aligned}$$

Marking scheme: 1 for answer. Simplification not necessary.

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Very short answer questions

1. 2 marks Each part is worth 1 mark. Please write your answers in the boxes. **Marking scheme:** 1 for each correct, 0 otherwise

   (a) Evaluate $\tan\left(\frac{3\pi}{4}\right)$.

   Answer: -1

   Solution:

   $$\sin(3\pi/4) = \frac{1}{\sqrt{2}} \qquad \cos(3\pi/4) = -\frac{1}{\sqrt{2}} \qquad \text{so} \qquad \tan(3\pi/4) = -1$$

   (b) Compute $\lim_{t\to 2} \sqrt{2t^3 - 16}$.

   Answer: 0

   Solution:

   $$\lim_{t\to 2} \sqrt{2t^3 - 16} = \sqrt{\lim_{t\to 2} 2t^3 - 16} = \sqrt{16 - 16} = 0$$

Short answer questions — you must show your work

2. 4 marks Each part is worth 2 marks.

   (a) Find all x such that $x^2 + 5x + 6 > 0$.

   Solution:

   $$x^2 + 5x + 6 = (x + 2)(x + 3)$$

   So the expression is positive for $x < -3$ or $x > -2$. **Marking scheme:** Give 1 mark for factoring. Give the second mark for correct intervals (written as inequalities, intervals or marked on the number line). Give 1 mark total if factoring is incorrect, but the intervals are correct for their factoring. Give 0 for answer without any work.

   (b) Compute the limit $\lim_{x\to -7} \frac{2x + 14}{x^2 - 49}$

   Solution: If try naively then we get 0/0, so we simplify first:

   $$\frac{2x + 14}{x^2 - 49} = \frac{2(x + 7)}{(x + 7)(x - 7)} = \frac{2}{x - 7}$$

Hence the limit is $\lim_{x \to -7} \frac{2}{x-7} = -\frac{2}{14} = -\frac{1}{7}$. **Marking scheme:** 1 for factoring+cancelling, 1 for answer. If answer with no working then 0.

Long answer question — you must show your work

3. 4 marks Compute the limit $\lim_{x \to -1} \frac{x+1}{\sqrt{x^2+15}-4}$.

Solution: If we try to do the limit naively we get 0/0. Hence we must simplify. **Marking scheme:** If multiply by correct conjugate then 2 marks. If multiply by something close to correct then 1. Else 0.

$$\begin{aligned}
\frac{x+1}{\sqrt{x^2+15}-4} &= \frac{x+1}{\sqrt{x^2+15}-4} \cdot \frac{\sqrt{x^2+15}+4}{\sqrt{x^2+15}+4} \\
&= \frac{(x+1)(\sqrt{x^2+15}+4)}{(x^2+15)-4^2} \\
&= \frac{(x+1)(\sqrt{x^2+15}+4)}{(x^2-1)} \\
&= \frac{(x+1)(\sqrt{x^2+15}+4)}{(x+1)(x-1)} \\
&= \frac{\sqrt{x^2+15}+4}{(x-1)}
\end{aligned}$$

Marking scheme: If correct simplification then 1 mark, else 0. So the limit is

$$\begin{aligned}
\lim_{x \to -1} \frac{x+1}{\sqrt{x^2+15}-4} &= \lim_{x \to -1} \frac{\sqrt{x^2+15}+4}{(x-1)} \\
&= \frac{8}{-2} \\
&= -4
\end{aligned}$$

Marking scheme: 1 for answer.