Exotic sheaves and actions of quantum affine algebras

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Exotic sheaves

Consider the (equivariant) derived categories of the (Grothendieck–Springer resolution). These categories are endowed with an action of the affine braid group. The *exotic t-structures* are the unique t-structures on these categories such that

1. the positive braids act right t-exact, and
2. the pushforward to the base is t-exact.

They were most famously used by Bezrukavnikov and Mirković [BM] to prove deep results from modular representation theory.

Categorical actions and braid groups

Typically:

- Exotic t-structures arise very naturally from categorical actions.
- They were most famously used by Bezrukavnikov and Mirković [BM] to prove deep results from modular representation theory.

Results: inducing exotic t-structures

For our purposes, we identify the weights with n-tuples \( \ell \) of integers (e.g. for \( \mathfrak{g}_0 \), the roots are \( a_1 = (-1, 1) \) and \( a_n = (1, -1) \)). We assume that \( \sum_k k = n \) and all \( k \geq 0 \). In particular, we get an affine braid group action on the central category \( \mathcal{H}(1, \ldots, 1) \).

Our viewpoint

Exotic t-structures arise very naturally from categorical actions.

Applications and future work

- Obtain exotic t-structures on spaces where the known constructions (exceptional sets, tilting) do not apply.
- Of particular interest: exotic t-structures on convolution varieties of the affine Grassmannian (see example on the right). We will expand this to more general convolution varieties in future work.
- We expect that structural results (weight structure, description of irreducibles) can be obtained with our method and be applied to these new examples.

The main example

Define the varieties

\[ \mathcal{Y}(\ell) = \mathcal{C}(z)^n \subset L_0 \subset \cdots \subset L_n \subset \mathcal{C}(z)^n : zL_i \subseteq L_{i+1}, \dim(L_i/L_{i+1}) = k_i \]

and

\[ \mathcal{G}_{\mathfrak{g}} = \mathcal{Y}(\ell) = \mathcal{C}(z)^n \subset L_0 \subset \cdots \subset L_n \subset \mathcal{C}(z)^n : zL_i \subseteq \mathcal{L}_i, \dim(L_i/L_{i+1}) = k_i \].

These convolution varieties are well-studied and used, for example, to categorify link invariants or give a (quantum) K-theoretic analogue of the geometric Satake equivalence. Note that \( \mathcal{Y}(1, \ldots, 1) \) has an open subvariety isomorphic to \( \mathcal{N} \) and the \( \mathcal{Y}(\ell) \) have open subvarieties isomorphic to partial Grothendieck–Springer resolutions.

Careful analysis of the structure and combinatorics of categorical \( \mathfrak{g}_\mathfrak{b} \)-actions and the associated braid group actions allows us to induce t-structures using the following lemma (which is based on a theorem of Polishchuk [P]).

Lemma. Let \( \Phi : D^b(X) \to D^b(Y) \) be a conservative Fourier–Mukai functor. Assume that we are given a t-structure on \( D^b(Y) \) such that \( \Phi \circ \Phi \) is right t-exact. Then there exists a unique t-structure on \( D^b(X) \) such that \( \Phi \) is t-exact.

How?

References


